Implicit Differentiation and the AP Exam

**Part 1 – What is Implicit Differentiation?**

When you first learned about finding derivatives, it was probably with functions defined “explicitly”, meaning that the y (or ) is defined very clearly for you in terms of “x”.

*Examples:*



However, some curves are defined “implicitly”, meaning that there are functions existing within the curve but you do not know clearly what they are. The x’s and y’s may be on both sides of the equation and are often even jumbled together.

*Example:*

The graph below corresponds to the curve given by 



We still need to be able to use derivatives to answer questions about the behavior of this curve. Although we could solve the equation for y, that would be an extra step, and in many equations we can’t solve for y. We need another technique!

*Implicit Differentiation*

When a curve is defined implicitly, you can find  using this method:



Note that the derivative has both x’s and y’s in it. That’s ok. You’ll see how this helps in the next part.

In summary, when you have an equation with x’s and y’s jumbled together you know that you have a candidate for implicit differentiation. Use the procedure outlined above to find your derivative.

**Part 2 – What Skills Will I need on the AP Calculus Exam?**

Here’s is what you’ll need to be able to do on the AP Calculus Exam

* Use implicit differentiation to find dy/dx
* Find points on a curve with particular slopes (including horizontal and vertical)
* Find a 2nd derivative implicitly and use it to answer questions
* Almost anything you could be asked to do with a regular derivative you could be asked to do with one you find implicitly.

**Example:**

Consider the curve given by (see the graph below)



1. Use implicit differentiation to find .
2. Find the points on the curve at which there are horizontal tangents by:

(a) setting 

(b) solving for “x”

(c) plugging your x value that you got for (b) back into the curve to get “y”.

1. Find the points on the curve at which there are vertical tangents using a similar procedure.
2. Find the points on the curve at which the tangent to the curve has a slope of .
3. Write the equations for the lines tangent to the curve at the points you found in #4.
4. Find  and use it to determine if the points where there are horizontal tangents are maximums, minimums or neither.

**Part 3 – Sample AP Problems – Multiple Choice and Free Response**

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* 1. ***Sample problems – Multiple Choice***

2003 #26



1998 #6



1997 #17



2008 #16



* 1. ***Sample Problems – Free Response***

2000 AB Free-Response #5



2008 AB Free-Response (Form b) #6



2004 AB Free-Response #4 (an old friend!)



2005 AB Free-Response (Form b) #5



Summary

*Here are some questions to think about. All of the answers can be found in this packet. Take this out again a little before the exam to refresh your memory.*

What is implicit differentiation and when do you use it?

What is the procedure for implicit differentiation?

What types of questions will you be asked on the AP Exam about implicit differentiation?